Homework 3
Perspectives on the Evolution of Structures
(Analysis 2 and GWB Study)

George Washington Bridge Structural Study and Analysis

In this assignment you will learn how to calculate the force and stress in the main span cable of a suspension bridge. You will also investigate how the overall form of the bridge affects this force, and lastly investigate some of the economic implications of the choice of form.

The pages following this assignment sheet contain some background on the George Washington bridge, as well as a description of how to compute the force in the main cables of the bridge. Read this material carefully, paying particular attention to the derivation of the equation $H = wL^2 / 8d$ that uses concepts of moment equilibrium.

Now, on to your assignment, the calculations of the behavior of the GWB:

1. For the George Washington Bridge, compute the tension force in each cable at the midspan due to combined dead and live load. Remember that there are four main cables (Fig 1) that equally share the work of carrying the load. Express your result in kips (1 kip = 1000 pounds).
2. Stress in a structural member carrying only tensile forces is the ratio of the amount of force to the cross sectional area of the member, that is stress = force / area. Calculate the stress in the main cables of the GWB. Express your result in kips per square inch (ksi). The diameter of the cable, needed to calculate the cross sectional, area can be found in the following pages of the structural study.
3. The allowable stress used in the design of the main cables is 100 ksi. Calculate the safety factor for the main cables, recalling that safety factor is the ratio of allowable stress to actual stress in the member, that is, safety factor = allowable stress / stress. Comment on your result. Is the safety factor close to one? Greater or less than one? Are you happy with this safety factor as a member of the public who might use this bridge?

The choice of the ratio of main span length (L) to cable sag (d) (See Fig. 1 for definitions) is a very important design choice in determining the engineering and aesthetic function of a suspension bridge. You will now investigate the aesthetic, and engineering implications of Othmar Amman's choice of span to sag ratio

$$R = \frac{\text{span}}{\text{sag}} = \frac{L}{d}$$

for the George Washington Bridge.

The Menai bridge has a cable sag of 46 feet and a main span of 584 feet, and the Verrazano Narrows Bridge has a cable sag of 453 feet and a main span of 4260 feet.

4. Calculate R for the GW, Menai, and Verrazano bridges.
5. If the span of the GWB remained constant, at 3500 feet, what would the cable sag have to be to give the GWB the same R value as the Menai bridge? For the Verrazano?

6. Draw sketches, to scale, of how the GWB would have looked with each of the three R ratios.
7. Calculate what the force would be in the main cables if the GWB had the R ratios of the Verrazano Gate and Menai bridges. Assume that the main span length and the loads are fixed, and that only the cable sag changes. How does R influence the cable force?
8. For the R ratio of the Verrazano bridge, calculate how much the cross sectional area of the cables would have to change to keep the stress constant (hint: the area and force are proportional if the stress is to be held constant).
9. Calculate what the diameter of the main cable would be in this case. Compare it to the diameter of the cables in the actual GWB. Do you think this difference is significant enough to have aesthetic implications for the bridge.

You must use graph or engineering paper for your calculations, and must staple all sheets together. Show all your work and place boxes around your answers.

Structural Study:

History and Influences

A great engineering debate raged along the Hudson River in the early twentieth century. Increased population densities and booming interstate commerce created new demands for a high capacity transportation network between New York and New Jersey. Both bridges and tunnels were proposed as potential solutions for new Hudson River crossings, although the decisions leading to their physical location, construction method, and total cost were formidable obstacles.

Othmar Ammann, a Swiss-trained engineer with American bridge building experience, had a solution. His proposal for a Hudson River crossing was a suspension bridge, the longest in the world at the time, which would carry vehicular traffic from the cliffs of the Palisades to 179th Street in New York City. Ammann separated himself from his competitors by eliminating heavy rail lines from the design criteria and by using a new mathematical theory to justify his record-breaking span. The George Washington Bridge was completed in 1931 and carried 5,000,000 vehicles across the Hudson River in its first year of operation. In 2011, the total traffic crossing the bridge was over 100,000,000 vehicles.

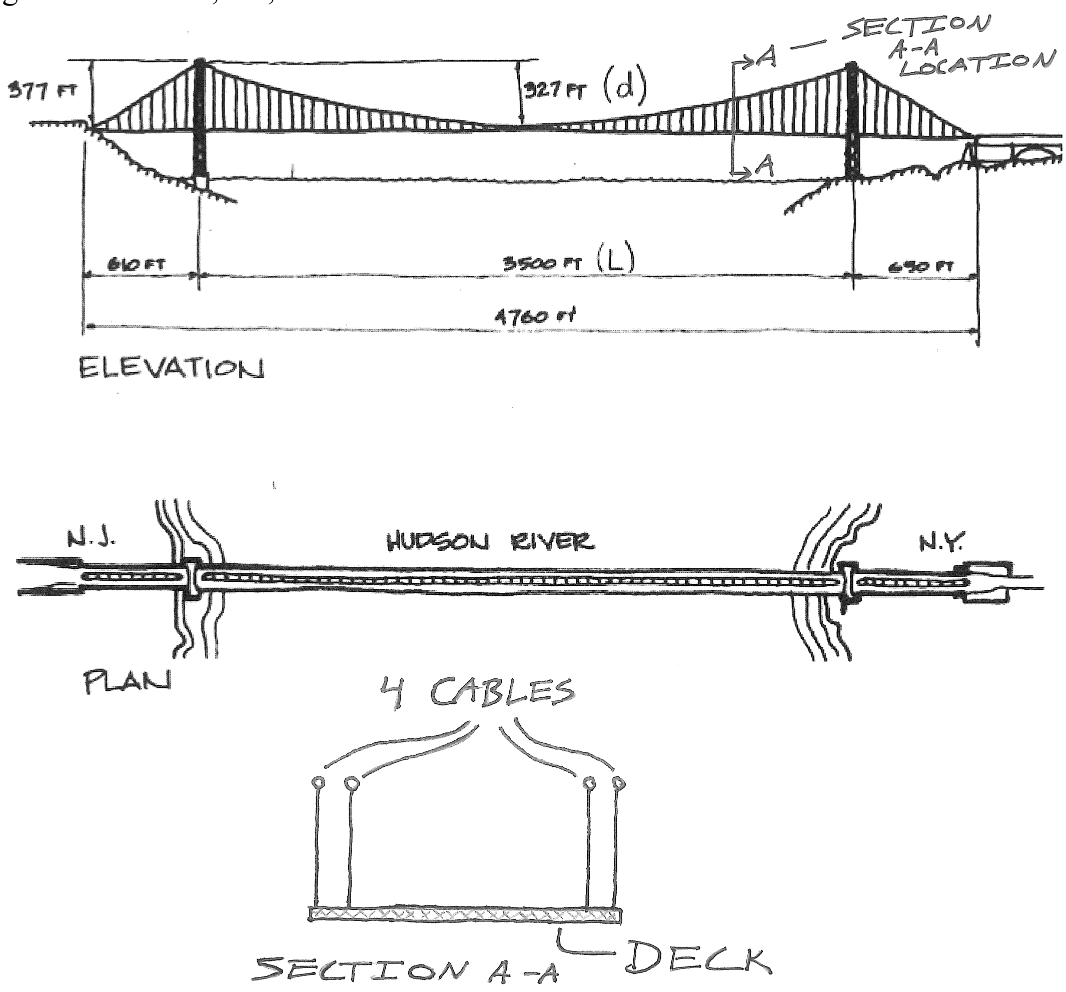


Figure 1: Elevation, plan, and section of George Washington Bridge

Structural Description

The George Washington Bridge crosses the Hudson River with two tall steel towers and a bridge deck suspended from 4 steel cables (figure 1). Each tower is 576 feet tall and consists of steel members connected in a truss-like framework. The original design intention was to encase the steel towers in reinforced concrete and then apply a masonry facade. The towers were instead left in their bare steel form after funds for the bridge ran low with the Great Depression of 1929.

Four main suspension cables span 3500 feet between towers. The flexible steel cables are supported at the top of each tower and anchor in heavy concrete blocks at both ends of the bridge. Each steel cable is 32 inches in diameter and is made up of 26474 parallel steel wires (figure 2).

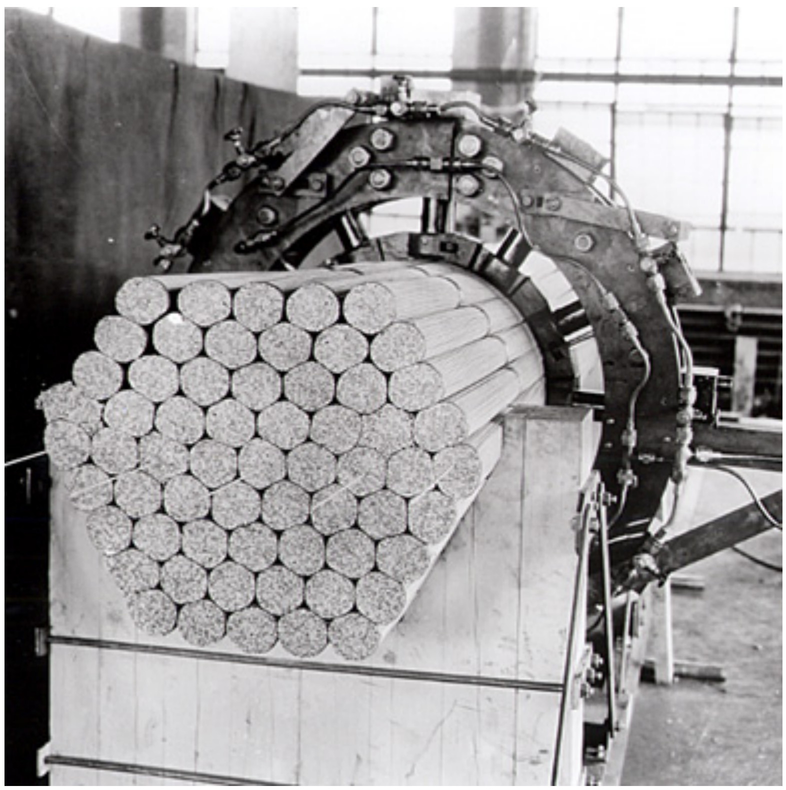


Figure 2: GWB main cable prior to compression to circular cross section. (http://www.panynj.gov/tbt/gwbhistorygallery.htm)

The bridge deck is 119 feet wide and is connected to the main cables by vertical steel suspenders. The original design carried six lanes of traffic supported by shallow floor beams. The vehicular capacity was doubled in 1962 with the completion of a truss-stiffened lower deck.

Design Loads

The dominant loads acting on the bridge are its own self weight, live load from vehicular traffic, and wind. The bridge deck and the steel cables are the main contributors to self weight considered in this study. We will approximate the self weight of the bridge with a uniformly distributed load of 39 kips/ft (figure 3).

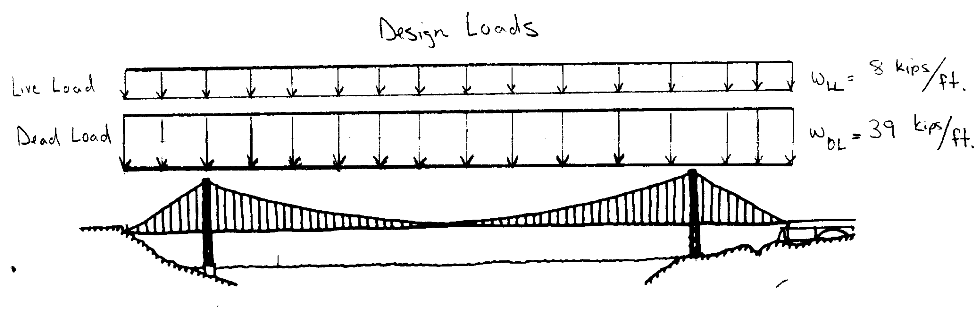


Figure 3: Design loads for GWB .

Vehicular traffic, and especially truck traffic, produces the highest live load forces on this bridge in service. It is improbable that all lanes of the bridge will be loaded from end to end with heavy trucks at the same time. Therefore, our chosen design live load reflects the expected truck traffic density, not the maximum number of trucks possible on the span. For this study, we will assume that trucks produce a uniformly distributed live load of 8 kips/ft (figure 3). Note that these are the total applied loads and are divided equally between the four main cables.

Since the effects of wind on the bridge are well controlled by Ammann's design, we will not consider wind as a design load in this study. Oscillatory deflections due to wind are prevented by the use of a wide, heavy bridge deck.

Analysis

The main goal of this study will be to understand how the main suspension cables carry gravity loads. We will begin our analysis by following the flow of gravity forces from the bridge deck to the suspension cables and finally to the tower foundations. Self weight and live load are applied at the bridge deck level as vertical loads (figure 3). These forces are then picked up by the steel suspenders and deposited onto the main suspension cables. The main cables are flexible and modify their geometry to convert the vertical forces into axial tension along their length (figure 4). The main cable's axial tension is converted back to a vertical force at the steel towers, where it is transferred to the ground at the tower foundations (figure 5).

Now that we have an idea of how the forces flow through the bridge, we can perform a simple calculation to estimate the axial force in a suspension cable at the middle of the span. We will find that the force in the cable at midpan depends on the length of the span and the ratio of the cable sag to the span length. The cable is horizontal at the midspan because of symmetry, and the value of this force can be determined using a free body diagram of half the bridge, and the concept of moment equilibrium. (figure 5).

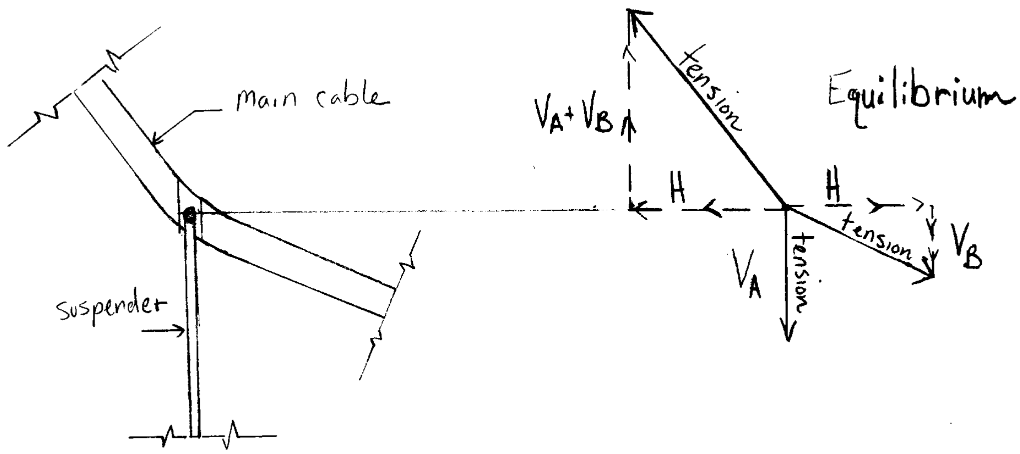
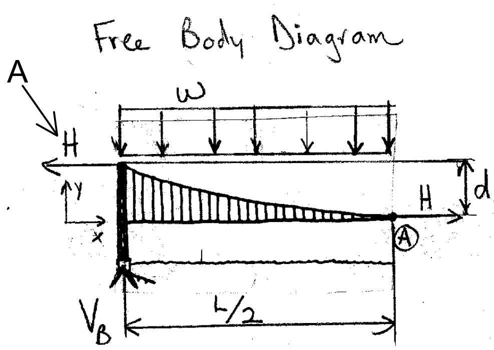


Figure 4: Cable and Force Geometry.



Moment equilibrium at A

$$\sum M_A = 0$$

$$-\frac{wL}{2}\frac{L}{4} + Hd = 0$$

$$H = \frac{wL^2}{8d}$$

Figure 5: Cable force calculation at midspan.

The cable force at midspan, therefore, is given by the simple equation $H = wL^2/8d$. This can also be expressed as

$H = \frac{wL}{8}R, R = \frac{L}{d}$.

In examining this equation, we can interpret w as representing the external loads acting on the bridge, L as representing the overall size of the bridge, and R as representing the shape of the bridge. Thus, the shape or form of the bridge, given by R, transforms an external force w into a force in the bridge, H. This is the mathematical expression of the concept that function follows form.

If the total loads as in Figure 3 are used in the above equation for *H* the force in each cable is Individual cable force $= H_{cable} = \frac{H}{4}$. The cable stress is $\sigma_{cable} = \frac{H_{cable}}{\text{cable cross section area}} = \frac{H_{cable}}{A_{cable}}$

and the safety factor (FS) is $FS = \frac{\text{allowable stress}}{\sigma_{cable}} = \frac{\sigma_{allowable}}{\sigma_{cable}}$